IJDI-ERET

INTERNATIONAL JOURNAL OF DARSHAN INSTITUTE ON ENGINEERING RESEARCH AND EMERGING TECHNOLOGIES
Vol. 12, No. 1, 2023
www.ijdieret.in

(Research Article)

Application of Galerkin Approach for the Determination of the Optimum Buckling Value of a Prismatic Bar

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Abstract

Structural instability of the most used engineering material steel under the action of axially applied compressive load which thus actuates its failure has necessitated its study inquest to understand the contributory factors to such failure by buckling and the behavioural pattern of the material. This study was therefore based on the application of Galerkin's approach of the method of weighted residuals to determine the optimum buckling (deflection) value of a prismatic bar made of AISI 8620 alloy steel material. The percentage composition of the elements present in the material was determined using atomic absorption spectroscopy (AAS) machine. The mechanical properties of the material such as its compressive yield strength, elastic modulus, tensile and compressive strengths were experimentally determined with the aid of an Amatez EZ 250 tensile and compressive testing machine according to ASTM. The essential variables such as the cross-section area, moment of inertia, slenderness ratio, optimum buckling load, bending moment, position of the neutral axis, radius of gyration and critical stress were all evaluated using their appropriate formula. Analytical formula for calculating the deflections of a beam was equally employed to validate the results of Galerkin method. From the results obtained, the Galerkin's and analytical methods employed gave an optimum buckling (deflection) value of 2.8607mm and the results of the analytical method correlated well with that of the Galerkin's procedure. The material lost stability within its elastoplastic boundary before its plastic deformation. Therefore, this determined optimum buckling (deflection) value of AISI 8620 alloy steel material gave well-correlated values and should be referenced during structural/mechanical design of systems to ensure their safety and optimal performance characteristics.

Keywords: Galerkin's method; mechanical properties; optimum buckling; prismatic bar; analytical method; steel.

1. Introduction

The rate of failure of structural and machine members often subjected to axial or longitudinal loads requires the attention of designers and further investigations to mitigate the effects posed by such failures. Most engineering designs, machines and structural members are made of steel; majorly alloy steel materials owing to its cost-effectiveness, availability, weldability, machinability and good mechanical properties [1-3]. Steel materials find application in axles, automobile wheels, camshafts, light-duty gears, gudgeon pins, spindles, feed rods, crankshafts, engine cylinders, etc. Also, in structural concepts (buildings), alloy steel is used with concrete reinforcements to boost its load-carrying integrity and another supportive member intrinsic to the building [4-6]. Hence, the study of the failure rate of alloy steel materials applied in diverse fields of engineering is very imperative.

The mechanical characteristic exhibited by steel is of a wide range of which the strength factor is the dominant property [7-9]. Although the behavioural pattern of steel is immensely affected by its chemical compositions, heat treatment, manufacturing method and operating environment, some mechanical properties determine the behavioural pattern of steel. These include the yield strength, ultimate strength, modulus of elasticity, Poisson's ratio and stability factor [10-12]. The design or structural engineer may centre their interest on these mechanical properties; however, these properties cannot be realistically obtained without a proper assessment of the chemical characterization of the steel [13-15].

The stability factor of steel is associated with its buckling integrity. Material failures caused by buckling often occur due to loss of structural stiffness. The stability of a material is connected with the position of its centre of gravity and the moment its weight exerts about an axis of reference [16-19]. This instability of steel under load conditions could be studied with relevant concern paid to the equilibrium situation of the steel material and the orientation of the steel- strut or column [20-22]. A structural or machine member that carries an axial compressive load is called the strut. If the strut is vertically

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ISSN 2320-7590 (Print) 2583-3863 (Online)

oriented, that is, inclined at 90° to the horizontal it is called a column [23-24]. Material equilibrium conditions are defined under three types; stable, neutral and unstable. As explained by Yamaguchi et al [25], in the state of stable equilibrium the steel structure (strut or column) would return to its initial shape undeflected when the acting external axial load is removed. Further in their explanation, the state of neutral equilibrium is one at which the limit of elasticity is reached. At such conditions, the steel structure would assume equilibrium at its new deflected position. In the case of unstable equilibrium, steel (strut or column) would tend to move farther away from its initial position maintaining a deflected state.

Buckling of steel (strut or column) was classified by Yang et al [26] and Li et al [12] into four types: global, distortional, local and interactive buckling behaviours or modes. Naderian & Ronagh [27] explained global buckling as a buckling mode in which the member has no deformation in its cross-sectional shape, consistent with classical beam theory. Local buckling was also defined as a buckling behaviour that involves plate-like deformation alone, without the translation of the intersection lines of adjacent plate elements. The buckling mode with cross-sectional distortion that involves the translation of some of the fold lines or intersection lines of adjacent plate elements is known as distortional buckling behavior [27-28] The interactive buckling mode according to Martin et al [29] was explained to be the coupled effects produced by the combination of global, local or distortional. The coupled interaction of distortional-global is a complex phenomenon that is complex and difficult to predict. As reported by Martin et al [30], the behavioural mode of distortional-global interaction can be altered depending on the nature of the global buckling mode: the distortional major-axis flexural-torsional (DMFT) and distortional minor-axis flexural (DMF). Sequel to this concern, Lazzari & Batista [23] stated that the deflection values of steel and other engineering materials are also useful tools in analyzing the average strength of structural and machine members since the two properties (deflection and strength of steel) are inversely related.

Due to the wide range of applications of steel material and the consequent distortional buckling behaviour leading to the failure of the material while in operation as a result of induced critical stress, it is therefore imperative to determine its optimum buckling deformational or deflectional value using the Galerkin approach of finite element method, and analytical methods. This study was focused on AISI 8620 alloy steel.

2. Materials and Methods

2.1 Materials and equipment: The base material used in this study was AISI 8620 grade of alloy steel material which was obtained from General Steel Mill, Asaba, Delta State. The specimen has a length of 200mm and a circular cross-section of diameter of 25mm. AAS was used to characterize the specimen- AISI 8620 to get its percentage elemental compositions according to Welz and Sperling, [31]. Amatez EZ 250 was used to carry out compressive and tensile tests on the specimen to ascertain the values of the needed mechanical properties that were employed in Galerkin's approach. Matlab software tool was used to solve the matrix equation obtained from the application of Galerkin's method.

2.2 *Methods:* The specimen was first cleaned and polished to avoid interference with the dirt during the experimental process. It was then placed in the AAS machine and the spark was introduced. The sparking was done 2-3 times after which the best result was taken. The tensile strength, compressive strength, compressive yield strength and modulus of elasticity of the AISI 8620 prismatic bar were experimentally determined using the tensile and compressive testing machine according to ASTM. Some material data and buckling variables that are essentially needed in the application of Galerkin's approach to determining the optimum buckling value of the test specimen are numerically computed, thus:

The geometric model used in this study is of circular cross-section, therefore, its moment of inertia expressed as:

$$I = \frac{\pi d^4}{64} \tag{1}$$

The area of the cross-section of the model is given as:

$$A = \frac{\pi d^2}{4} \tag{2}$$

The radius of gyration (K) is computed using the formula expressed as:

$$K = \left(\frac{I}{A}\right)^{1/2} \tag{3}$$

The slenderness ratio (λ) is given as:

$$\lambda = \frac{L}{K} \tag{4}$$

For a model with one fixed end and one free end, its equivalent length, $l_e = 2l$ and the buckling/critical load formula as:

$$P = \frac{\pi^2 EI}{{l_e}^2} = \frac{\pi^2 EI}{4L^2} \tag{5}$$

The critical buckling stress computational formula was given by Partskhaladze et al [32]:

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2} \tag{6}$$

The bending moment (M(x)) can be expressed as the maximum compressive stress of a beam subject to axial compression as:

$$\sigma_c = \frac{P}{A} + \frac{Mc}{I} = \text{critical stress} + \frac{Mc}{I} \quad (7)$$

Where σ_c is maximum compressive stress of a beam subject to axial compression, P is critical load, A is cross-sectional area, I is moment of inertia of the section, and c is dimension representing the position of the neutral axis of the section.

The position of the neutral axis of the material (c) expressed as:

$$c = \frac{\text{depth of the section}}{2} = \frac{\text{diameter of the section}}{2} \quad (8)$$

Table 1 shows the summary values of some buckling variables and material data employed in Galerkin's approach. Galerkin's methodical procedure expressed by Alavala [33] was employed to determine the optimum buckling value of the material.

Table 1. Summary values of material data and buckling variables

SN	Parameters	Values
1	Moment of inertia	$1.9175 \times 10^{-8} m^4$
2	Cross-sectional area	$4.9087 \times 10^{-4} m^2$
3	Radius of gyration	$6.2501 \times 10^{-3} m$
4	Slenderness ratio	31.9995
5	Position of neutral axis	0.0125m

2.2.1 Determination of elements equations using Galerkin's method: The mathematical model used in Galerkin's approach is given as:

$$\int_{D_1}^{D_2} RN_i dD = 0 \quad (9)$$

Where R is residual functional, N_i is shape functional, D_1 *and* D_2 are solution domains.

Employing Timoshenko's beam model or Euler-Bernoulli's beam model given in equation 10:

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI} \quad (10)$$

The residual functional, R is given as:

$$R = \frac{d^2y}{dx^2} - \frac{M(x)}{EI} \quad (11)$$

Substituting residual functional into equation 9:

$$\int_{D_1}^{D_2} \frac{d^2y}{dx^2} N_i dx = \int_{D_1}^{D_2} M(x) N_i dx \quad (12)$$

By applying integration by parts on equation 12 and solving for element 1 having nodes 1 and 2:

$$\frac{1}{10}(-y_1 - y_2) = \frac{dy}{dx}(x_1) + \frac{5M(x)}{EI} \quad (13)$$

$$\frac{1}{10}(y_1 - y_2) = -\frac{dy}{dx}(x_2) + \frac{5M(x)}{EI} \quad (14)$$

Putting equations 13 and 14 in matrix form gives:

$$\begin{bmatrix} \frac{-1}{10} & \frac{1}{10} \\ \frac{1}{10} & \frac{-1}{10} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{dy}{dx}(x_1) \; + & \frac{5M(x)}{EI} \\ \frac{-dy}{dx}(x_2) & \frac{5M(x)}{EI} \end{bmatrix} \quad (15)$$

The element topology assists in the computation and assembly of elements equations. Through the application of the element topology definition and concept of nodal continuity, other element equations are obtained and assembled.

Based on the classical report of Camotim et al [1] and Bebiano et al [2], the analytical equation for calculating the deflections of a beam at distinct nodal positions is given as:

$$y_x = A \sin\left(\frac{\pi x}{L}\right) \quad (16)$$

Where y_x = nodal deflection point being considered, A is maximum deflection, x is the position of the node being considered, and L is length of the beam.

This equation of beam deflections was applied in this study to analytically determine the buckling (deflections) of AISI 8620 steel.

3. Results and Discussion

3.1 AAS analysis: Table 2 shows the percentage elemental composition of AISI 8620 alloy steel that was determined using the AAS machine.

Table 2. Chemical analysis of AISI 8620 alloy steel

Element	C	Si	Mn	Mo	S
Avg. content	0.200	0.165	0.750	0.158	0.0390
Element	Cr	Ni	Fe	P	
Avg. content	0.450	0.470	97.886	0.0295	

From Table 2, it can be seen clearly that AISI 8620 alloy steel contains chiefly iron (Fe) to a percentage of 97.886% and a total of 2.2615% of the other constituents. The result of the chemical analysis presented in Table 2 matched the standard.

3.2 Mechanical test: The values of the tensile strength, compressive strength, yield strength (compressive) and Young's modulus of elasticity of the steel material that was determined experimentally are shown in Table 3.

Table 3. Experimentally determined values of the mechanical properties of AISI 8620 steel

Property	Value
Tensile strength	530MPa

Compressive strength	510MPa
Modulus of elasticity	205GPa
Yield strength (Compressive)	385MPa

3.3 Materials buckling parameters: Table 4 shows the summary values of the required parameters that were determined numerically using their appropriate formula and are essential in the determination of the optimum buckling value of AISI 8620 steel.

Table 4. Materials buckling parameters

Parameter	Value
Optimum buckling load (P)	242.5KN
Critical stress (σ_{cr})	1.976GPa
Bending moment [M(x)]	2.249kNm

3.4 Optimum Buckling of AISI 8620: The deflections produced on the prismatic bar at the optimum buckling load condition of value 242.5kN using Galerkin's approach are shown in Figure 1. The deflections were considered at various nodal positions of the prismatic bar.

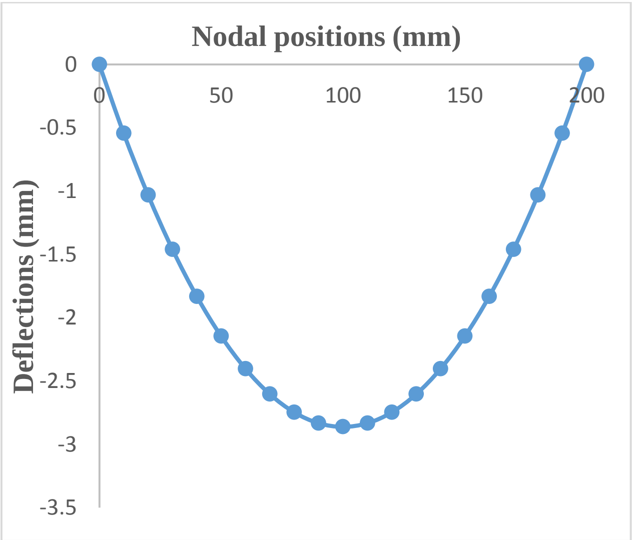

Figure 1. Deflections at various nodes of the prismatic bar

From Figure 1, the optimum deflection of the prismatic bar occurred at the central position of the material (100mm length of the bar). The optimum buckling value gotten at this point of the bar was 2.8607mm. Therefore, at this deflection value of the material, the material has become inelastic or non-linear. Figure 2 shows the buckling deflections of the material that was determined through the application of analytical technique.

From Figure 2, it could be seen that the optimum buckling deflection of the prismatic bar occurred at the centre of the bar with a value approximately the same as that obtained using Galerkin's method. To compare the results of both Galerkin's and analytical methods, Figure 3 is presented.

The obtained buckling deflections of the material using both approaches could be vividly seen to be accurate since their respective deflection values are well correlated. This, therefore attests to the correctness of the applied Galerkin's method in determining the optimum buckling deflection of a prismatic bar.

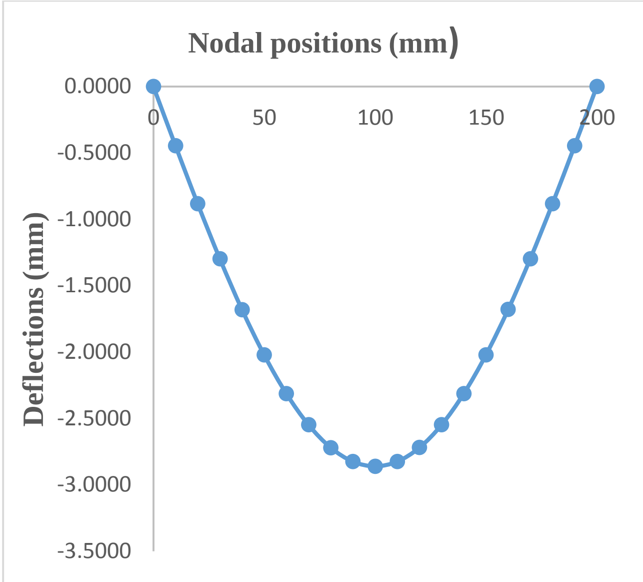

Figure 2. Analytical computation of buckling deflections of the material

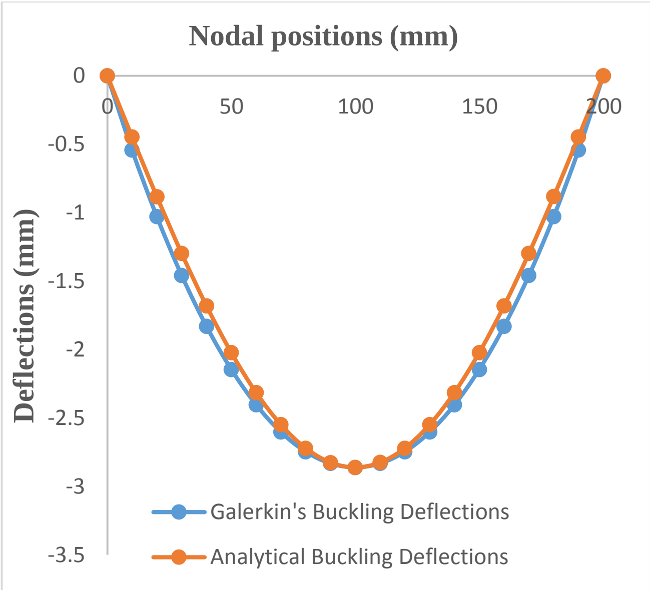

Figure 3. Comparison between buckling deflection values computed using Galerkin's and analytical methods

4. Conclusions

The structural stability of AISI 8620 alloy steel was studied to determine the optimum buckling (deflection) of the steel material by employing Galerkin's, and analytical methods. The optimum buckling load that was computed with the aid of

Euler's formula for fixed-free end condition was 242500N. The analytical method results gave an optimum buckling deflection of 2.8607mm. The results of the analytical formula that was used to calculate the buckling deflections of the prismatic bar had a very good correlation with that of Galerkin's method. In terms of failure criterion theories, this is the most favourable load, beyond which the structural instability of the material becomes very severe as attested by the various mode shapes and their corresponding load multipliers. The compressive yield strength of the material which was experimentally determined as 385MPa was exceeded to a value of 478MPa. The transitive behaviour of the material from linear to non-linear showed a clear bifurcation point before the commencement of the non-linear behaviour of the material. Therefore, the two methods applied in this study to determine the optimum buckling (deflection) value of AISI 8620 alloy steel material gave a well-correlated value and should be referenced during structural/mechanical design of systems to ensure their safety and optimal performance characteristics.

Nomenclature

AAS- Atomic absorption spectroscopy
DMFT- Distortional major-axis flexural-torsional
DMF- Distortional minor-axis flexural
I- Moment of Inertia
K- Radius of gyration
λ- Slenderness ratio
P- Buckling/critical load
R- Residual functional
σ_{cr}- Critical buckling stress
σ_c- Maximum compressive stress
y_x- Nodal deflection
N_i- Shape functional

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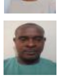

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