

(Research Article)

Non-Linear Regression Modelling of the Compressibility Properties of Soil in Calabar Municipal, Cross-River State

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Abstract

This work aims at studying the compressibility properties of soil in Calabar Municipal, Cross River State, Nigeria. Also, these properties are correlated with the Atterberg Limits of the soil. The magnitude of the overall compressibility prediction of regression model depends on several variable such as the compressive index (C_c), coefficient of consolidation (C_v), coefficient of compressibility (A_v), and coefficient of volume compressibility (M_v), which are determined by a consolidation test in the laboratory, by carrying out oedometer test. However, the test itself is time consuming and involves lots of calculation hence, the non-linear regression model of this research focuses on determining the relationship between the compressibility characteristics of the soil and its Atterberg Limits. The regression models developed for the ten samples using the Wolfram Application would enable for prediction of the different Compressibility Properties of the soil in Calabar Metropolis relative to its Atterberg Limits thereby reducing rigorous time-consuming activities. The regression models developed were observed to have a Coefficient of Determination (R^2) value above 90% which indicated the adequacy of the model to be employed for prediction. R^2 for LL, PL and PI are of values 0.955692, 0.951884 and 0.944464 respectively on M_v , the R^2 for LL, PL and PI are 0.97744, 0.976234 and 0.971964 on the C_v . Also, models for C_c gave an R^2 with values 0.990806, 0.99053 and 0.990396 respectively. Finally, A_v on the LL, PL and PI have R^2 as 0.969628, 0.965995 and 0.963419 respectively.

Keywords: Regression, Compressibility, Atterberg Limits, Consolidation, Coefficient of Determination.

1. Introduction

The Engineering sector of the country is beginning to experience an uprising in construction activities across various states. For States in Nigeria like the Cross-River State which is located in Niger Delta region of country and renowned for its different tourist sites, has seen engineering activities heightened. Most areas of the Niger Delta are usually characterized as marshy or swampy and soils predominant in this region are usually formed from vegetative decomposition in water which are classified as peat. Furthermore, these soils are usually loose and compressible. As such, pose problems for engineering construction of Structures or other facilities supported by soils. Similarly, in a bid to combat this engineering problems, understanding the various characteristics of a soil and ascertaining which properties would enhance or support an engineering construction is of great importance. Soils exist in various forms and are usually peculiar based on its mode of formation [1]. A given soil possess different Engineering properties.

These properties are usually taken into consideration during engineering activities involving soils, [2] [3]. This study basically seeks to establish an understanding of the compressibility properties of soil within Calabar Municipal, Cross-River State and generate models suitable for prediction of these compressibility parameter. The various compressibility parameters such as Coefficient of Compressibility (A_v), Compression Index (C_c), Coefficient of Consolidation (C_v), Coefficient of Volume of Compressibility (M_v), and Coefficient of Consolidation (C_c) are identified and evaluated in accordance with BS 1377 [4].

Compressibility is simply the property of a soil to undergo volume change when subjected to mechanical deformation. These volume reduction under compression is accompanied by an expulsion of water and void from the soil mass. The structure settles as a result of the soil mass's compression, which also causes the mass's volume to shrink. It is logically possible to determine the displacement that occurs at each particular soil mass boundary by adding up the strains imposed on by a change in the stress system. The compression of the soil mass caused by the applied forces might happen very instantly or gradually, depending on the permeability properties of the soil, [5]. It is

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necessary to determine the compressibility parameters of soils for safe and economic design of civil structures.

In order to calculate the consolidation settlement of normally consolidated and over consolidated saturated fine-grained soils, the compressibility parameters are determined by means of laboratory oedometer test on undisturbed samples based on Terzaghi's consolidation theory. These parameters can be influenced from the quality of samples used in the tests [6], [7], [8].

The total settlement of a structure occurs mainly in three forms which includes Immediate (elastic) Compression, Primary Consolidation and Secondary Consolidation (creep). Elastic settlement occurs immediately due to applied load without any change in the water content. Primary consolidation is a main component of settlement of fine-grained saturated soils having low permeability, because the excess pore water pressure dissipates with time. The secondary consolidation is creep of soils such as peats and soft organic clays under constant effective stress. The primary and secondary consolidation settlements can be several times greater than the elastic settlement in saturated fine-grained soils. [8], [9], [10], [11].

1.1 Justification of the study: Understanding the compressibility characteristics of the soil in the Calabar metropolis will be made possible by this research. Similarly, the non-linear regression model developed in this research will aid the determination of the relationship between the compressibility properties and the index properties thereby,

enabling Engineers to predict a sort after Compressibility parameter of soil in other areas of Calabar metropolis.

2. Methodology

2.1 Brief description of the study area: Calabar is the capital of Cross River State in the southern region of Nigeria, originally called "Akwa Akpa" in Efik language. The city is in close proximity with the Calabar and Great Kwa Rivers, as well as creeks of the Cross River (from its inland delta). The city is majorly recognised as Nigeria's tourism capital, owing to various initiatives that have made the city blossom in infrastructural and environmental development with land mass of about 406 square kilometers [2]. Calabar has a tropical monsoon climate characterised with long rainy season and little dry periods. Average high temperatures often range from 25 to 28 degrees Celsius throughout the year, with temperatures remaining quite stable and an annual average precipitation over 3,000 millimeters.

Soil samples were randomly gathered around the Calabar Municipality zone and the corresponding coordinates are established, aided by a Global Positioning System (GPS). Ten (10) soil samples were collected from ten (10) different boreholes with the depth not more than one meter. The representative samples were collected using spade and hand auger and further transported to the laboratory for testing in airtight bags to prevent loss of moisture. Each sample bag was identified with tags signifying a particular trial pit. Table 1 gives a description of the sites and the corresponding global positioning system coordinates.

Table 1. Boreholes site description and coordinates [2]

Trial pit	Description of the site	Coordinates	
1	Airport road by bye pass round about	4.97311717	8.35139460
2	Ayade industrial park avenue	4.99414897	8.36083792
3	Parliamentary extension by fly over	5.02230713	8.36442816
4	Parliamentary extension by UD king	5.01048990	8.34724997
5	Highway by MCC	4.98717878	8.33308365
6	International conference center road	5.03117557	8.33166806
7	Technical college roundabout	5.04511306	8.35785800
8	Ediba lane by old Ikang	4.97227068	8.33711635
9	EkongIta by marian road	4.96564815	8.33810129
10	Big Qua by Ndidem palace	4.97095135	8.33127767

2.2 Soil classification procedures: Given that soils are usually distinct and possess different peculiarities, it is essential to ascertain its properties and how it impacts or influences its usage for construction processes. Therefore, experimental test procedures were carried out in compliance with BS 1377:1990 in order to ascertain the various index and engineering properties of the soil. Classification techniques were also employed to delineate the different soil sample into soil groups they fall into using the notable American Association of State

Highway Transport Officials (AASHTO) [12] system of classification and the Unified Soil Classification System (USCS) [13]. Hence, tests conducted which includes Particle Size Distribution (Sieve Analysis Test), Atterberg Limits Test (Liquid limits, Plastic limits, Plasticity Index) enabled for the soil classification [14] [15]. Other test carried out are the Natural Moisture Content Test and Specific Gravity of Solids Test. The results outlining the Index properties of the soil and classifications is tabulated in Table 2 and Table 3 respectively.

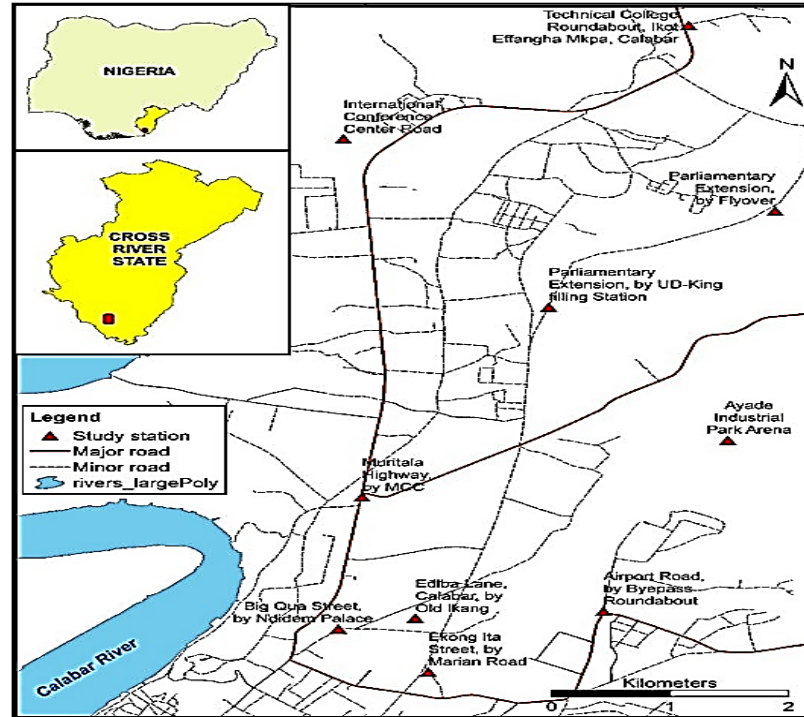


Figure 1. Map showing the study area

Table 2. Index properties of the soil [2]

Trial pit	Liquid limit LL (%)	Plastic limit PL (%)	Plasticity index PI	Specific gravity
TP1	38	25	13	2.6
TP2	36	22	14	2.3
TP3	30.5	18.5	12	2.5
TP4	38	27	11	2.9
TP5	33.5	20.5	13	2.5
TP6	35	22	13	2.6
TP7	37	23	14	2.3
TP8	38	25	13	2.9
TP9	33	21	12	2.5
TP10	35	23	12	2.9

Table 3. Soil classification [2]

Sample n0	% Passing sieve		Atterberg limits			Classification		Remarks
	N ₀ 40	N ₀ 200	LL	PL	PI	AASHTO	USCS	
1	68	10	38	25	13	A – 2 – 6	SC	Perfect materials
2	66	8	36	22	14	A – 2 – 6	SC	Perfect materials
3	68	6	30.5	18.5	12	A – 2 – 6	SC	Perfect materials
4	63	6	38	27	11	A – 2 – 6	SC	Perfect materials
5	62	4	33.5	20.5	13	A – 2 – 6	SC	Perfect materials
6	68	10	35	22	13	A – 2 – 6	SC	Perfect materials
7	64	8	37	23	14	A – 2 – 6	SC	Perfect materials
8	60	6	38	25	13	A – 2 – 6	SC	Perfect materials
9	64	6	33	21	12	A – 2 – 6	SC	Perfect materials
10	62	4	35	23	12	A – 2 – 6	SC	Perfect materials

2.3 Consolidation procedures: The consolidation test was employed to estimate the rate of settlement and the total amount of settlement of the soil layer under an applied load, the procedure and analysis is restricted to problems involving saturated soil mass which are principally clays and other soil soils of low permeability. Prediction of the settlement of a structure founded on a soil by the use of a laboratory consolidation test often requires that the sample used should be as nearly identical and representative of the soil mass. The consolidation test was conducted in the Laboratory of the Cross-River State University of Technology (CRUTECH) using the oedometer apparatus.

The soil sample is cut and trimmed so as to fit into a special metal ring provided for the test. The sample is normally a disc 75-100mm in diameter and 25mm thick. Porous disc is replaced on top and beneath the specimen and the assembled sample discs and ring are placed into a loading unit. A compressive load is applied and the changes in the thickness of the sample are read at the same time intervals. After settlement is complete usually after 24hours, the applied load is doubled, and readings are taken as before. This procedure is followed through several load increment.

The Consolidation test results are plotted in a graph illustrating the sample's compressive behaviour throughout the loading sequence. As the sample gets loaded, the air voids will slowly decrease, and water will escape. The graph is typically plotted showing the variation of the void ratio (e), with the corresponding changes in applied pressure on a semi

logarithmic graph where void ratio (e) is represented on the arithmetic scale and pressure on the log scale. The consolidation test helped to establish values for the different compressibility parameters of the soil and was carried out in accordance with BS 1377.

2.4 Regression procedures: Regression is simply an analytical method employed to statistical measure the relationship or association of one or multiple independent variables with a dependent variable. Similarly, regression models are formulated to enable the prediction of values for a dependent variable when the values of the independent variable are known and plugged into the model equation. The regression equation usually exists as linear or Non-Linear equations. [16], [17].

In this study, the Compressibility parameters of the soil which are sort after (to be predicted by the model equation) are represented as the Dependent variables while the Atterberg Limits are represented as Independent the variables.

The regression seeks to associate the Compressibility parameters of the soil with its Atterberg limits such that values for the Atterberg limits can be employed in the model equation to obtain values for the Compressibility parameters. Table 4 outlines the composition of the Independent and Dependent Variables. The study associates an individual Atterberg Limit parameter with an individual Compressibility parameter. The regression was computed using the Wolfram application and the resulting plots from each regression are presented

Table 4. Regression variables

Independent variables (X) Atterberg limits parameters	Dependent variables (Y) Compressibility parameters
Liquid Limit – LL	Coefficient of Consolidation - C_v
Plastic Limit – PL	Coefficient of Compressibility - A_v
Plasticity Index - PI	Coefficient of Volume Compressibility - M_v
	Compression Index - C_c

3. Results and Discussions

3.1 Compressibility characteristics: The study conducted a one-dimensional consolidation test to determine the compressibility parameters and the value for each parameter is obtained employing computational methods from Terzaghi and Peck. [11]. The correlation of the compressibility characteristics of soils to its Atterberg Limits (Liquid limit, Plastic limit and Plasticity Index) have been described graphically. Hence, the outcome of the result helps the geotechnical engineers to do quick identification or estimation of the compressibility parameters of soil. Compressibility parameters are used to predict how much settlement will take place under different structural loads.

3.2 Regression models: For each Test pit in the en locations, a regression model with interaction was created. Every sample's

initial regression model starts empty in the first phase of the procedure. Only the dependent variables, such as the M_v , C_c , A_v , and C_v enter the regression model at each result while the current independent variables in the equation are kept constant. The developed regression models (prediction expressions) are shown in the Table 5. Strength of the prediction parameters are documented, including values for R^2 and adjusted R^2 .

An R^2 value of 1.0 indicates a flawless prediction. The model generated for the soil samples shows that the value of R^2 with LL, PL and PI are constants of 0.955692, 0.951884 and 0.944464 respectively on M_v variable, indicating the adequacy of the model. While the R^2 for LL, PL and PI are 0.97744, 0.976234, and 0.971964 respectively on C_v . Then the regression model for C_v are 0.990806, 0.99053 and 0.990396 respectively, which gave higher values. Finally, the regression mode for A_v on the LL, PL and PI are 0.969628, 0.965995 and 0.963419

respectively, the models are seen as perfect model. The summary of the model result which gives results for the actual and predicted values of the various compressibility parameters

are tabulated in Table 7 through to Table 9 while the graphical plots are presented in Figure 2 through to Figure 13.

Table 5. Summary of the model result

Compressibility parameters with atterberg limits	R ²	R ² Adjusted	Regression equation
M _V LL	0.955692	0.944615	M _V = 0.00364999e ^{-0.050866 LL}
M _V PL	0.951884	0.939855	M _V = 0.00169333e ^{-0.0454005 PL}
M _V PI	0.944464	0.93058	M _V = 0.00116173e ^{-0.0510623 PI}
C _V LL	0.97744	0.971801	C _V = 0.000215056e ^{-0.0323186 LL}
C _V PL	0.976234	0.970293	C _V = 0.000137259e ^{-0.0306047 PL}
C _V PI	0.971964	0.964955	C _V = 0.000101356e ^{-0.0306516 PI}
C _C LL	0.990806	0.988508	C _C = 0.701528e ^{-0.0109543 LL}
C _C PL	0.99053	0.988162	C _C = 0.576118e ^{-0.00839981 PL}
C _C PI	0.990396	0.987996	C _C = 0.592438e ^{-0.0172066 PI}
A _V LL	0.969628	0.962034	A _V = 0.0111952e ^{-0.0405104 LL}
A _V PL	0.965995	0.957493	A _V = 0.00589134e ^{-0.0348444 PL}
A _V PI	0.963419	0.954273	A _V = 0.00655489e ^{-0.070582 PI}

Table 6. Actual and predicted values of coefficient of volume compressibility [2]

Sample no	Actual values	Atterberg limits			Predicted values		
	M _V	LL	PL	PI	M _V		
					LL	PL	PI
1	0.00053	38	25	13	0.00053	0.00054	0.0006
2	0.00043	26	22	14	0.00059	0.00062	0.00057
3	0.00084	30.5	18.5	12	0.00077	0.00073	0.00063
4	0.00056	38	27	11	0.00053	0.0005	0.00066
5	0.00053	33.5	20.5	13	0.00066	0.00067	0.0006
6	0.00059	35	22	13	0.00062	0.00062	0.0006
7	0.00068	37	23	14	0.00056	0.0006	0.00057
8	0.00068	38	25	13	0.00053	0.00054	0.0006
9	0.00086	33	21	12	0.00068	0.00065	0.00063
10	0.00038	35	23	12	0.00062	0.0006	0.00063

Table 7. Actual and predicted value of coefficient of consolidation [2]

Sample no	Actual values	Atterberg limits			Predicted values		
	C _V	LL	PL	PI	C _V		
					LL	PL	PI
1	0.000037	38	25	13	0.000063	0.000064	0.000068
2	0.000076	36	22	14	0.000067	0.00007	0.000066
3	0.000078	30.5	18.5	12	0.00008	0.000078	0.00007
4	0.000066	38	27	11	0.000063	0.00006	0.000072
5	0.000066	33.5	20.5	13	0.000073	0.000073	0.000068
6	0.000066	35	22	13	0.000069	0.00007	0.000068
7	0.000066	37	23	14	0.000065	0.000068	0.000066
8	0.000077	38	25	13	0.000063	0.000064	0.000068
9	0.000078	33	21	12	0.000074	0.000072	0.00007
10	0.000077	35	23	12	0.000069	0.000068	0.00007

Table 8. Actual and predicted values of compression index [2]

Sample no	Actual values	Atterberg limits			Predicted values		
	C _c	LL	PL	PI	C _c		
					LL	PL	PI
1	0.486	38	25	13	0.463	0.477	0.474
2	0.485	36	22	14	0.473	0.479	0.466
3	0.535	30.5	18.5	12	0.502	0.493	0.482
4	0.436	38	27	11	0.463	0.459	0.49
5	0.432	33.5	20.5	13	0.486	0.485	0.474
6	0.389	35	22	13	0.478	0.479	0.474
7	0.445	37	23	14	0.468	0.475	0.466
8	0.534	38	25	13	0.463	0.477	0.474
9	0.534	33	21	12	0.483	0.483	0.482
10	0.486	35	23	12	0.478	0.475	0.482

Table 9. Actual and predicted values of coefficient of compressibility [2]

Sample no	Actual values	Atterberg limits			Predicted values		
	A _v	LL	PL	PI	A _v		
					LL	PL	PI
1	0.0013	38	25	13	0.0024	0.003	0.003
2	0.0026	36	22	14	0.0026	0.003	0.002
3	0.003	30.5	18.5	12	0.0033	0.003	0.003
4	0.0026	38	27	11	0.0024	0.002	0.003
5	0.0026	33.5	20.5	13	0.0029	0.003	0.003
6	0.0029	35	22	13	0.0027	0.003	0.003
7	0.0026	37	23	14	0.0025	0.003	0.002
8	0.0026	38	25	13	0.0024	0.003	0.003
9	0.003	33	21	12	0.0029	0.003	0.003
10	0.0036	35	23	12	0.0027	0.003	0.003

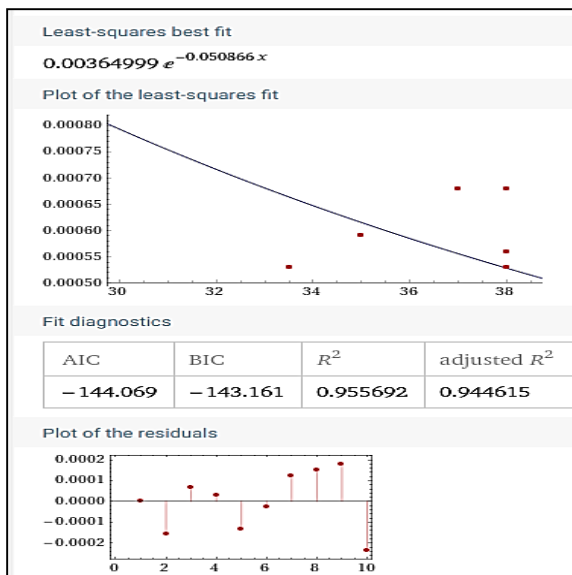


Figure 2. Regression plot of m_v with liquid limit

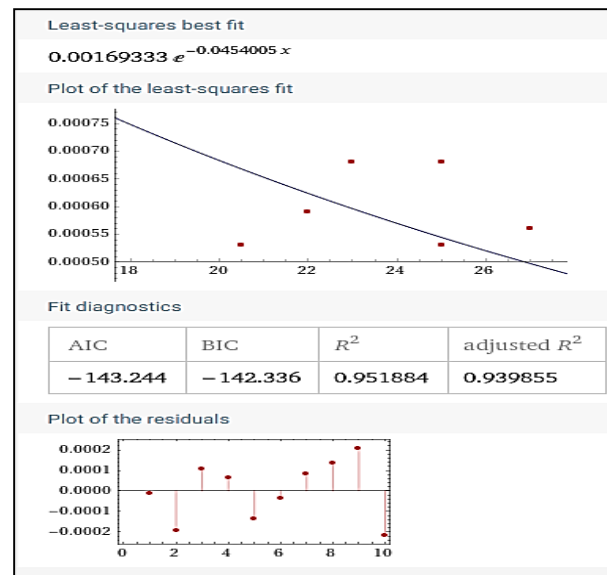


Figure 3. Regression plot of m_v with plastic limit

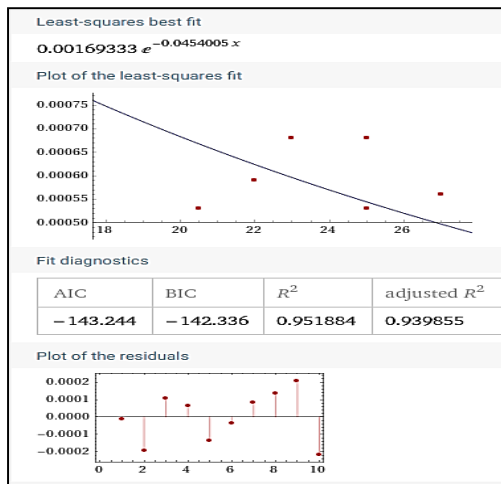


Figure 4. Regression plot of mv with plastic index

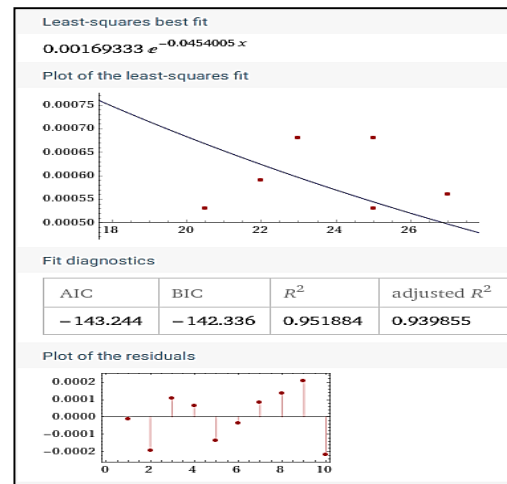


Figure 5. Regression plot of C_v with liquid limit

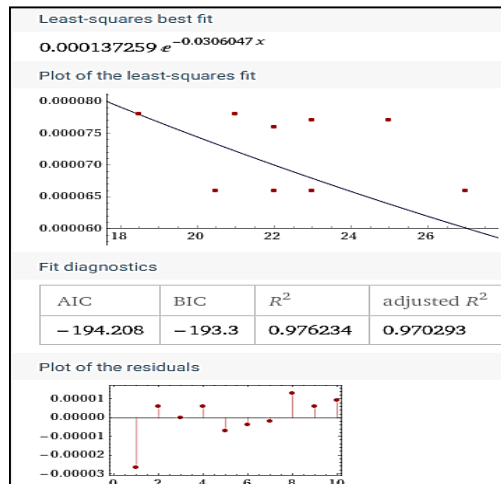


Figure 6. Regression plot of C_v with plastic limit

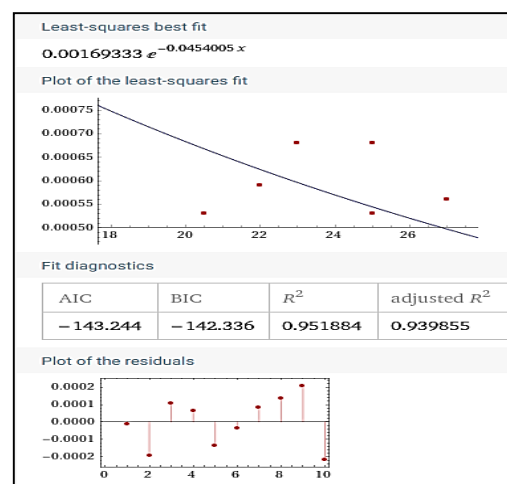


Figure 7. Regression plot of C_v with plastic index

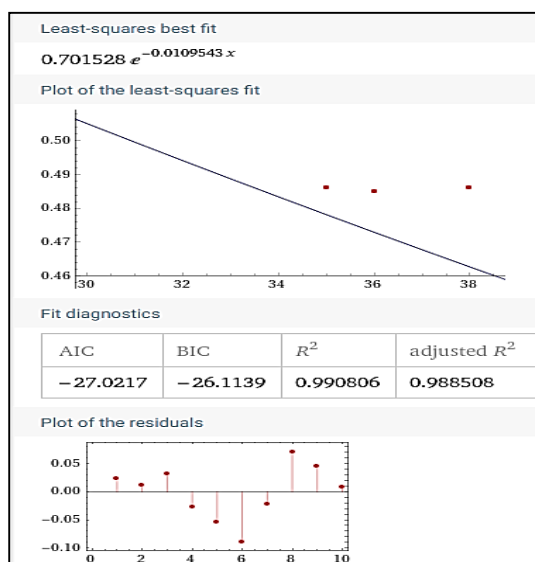


Figure 8. Regression plot of C_c with liquid limit

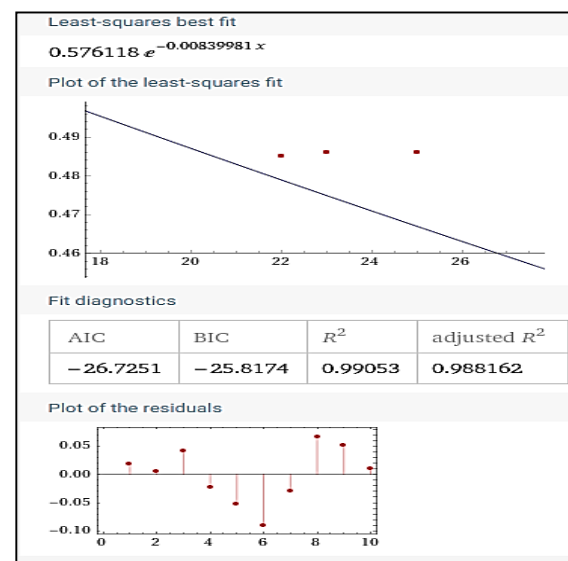


Figure 9. Regression plot of C_c with plastic limit

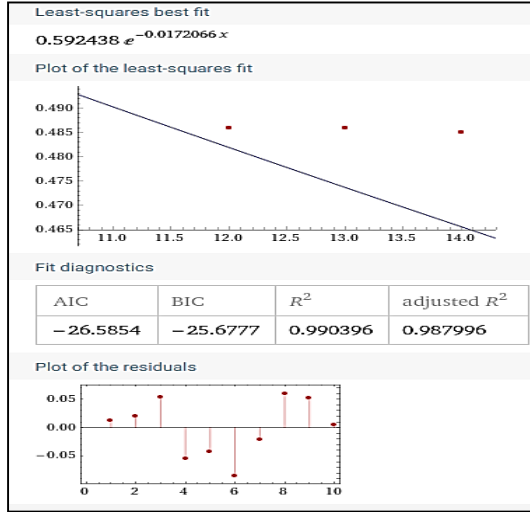


Figure 10. Regression plot of C_c with plastic index

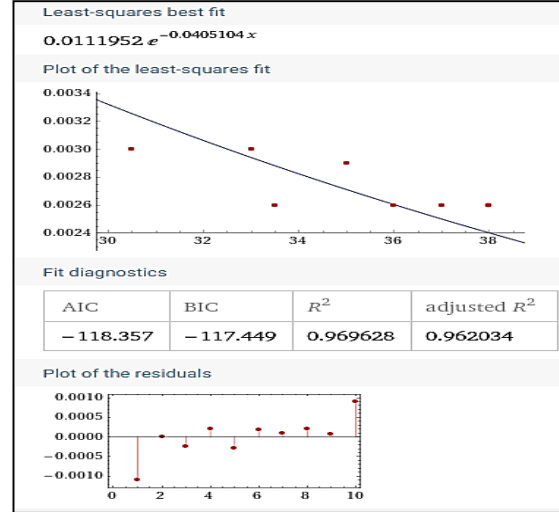


Figure 11. Regression plot of A_v with liquid limit

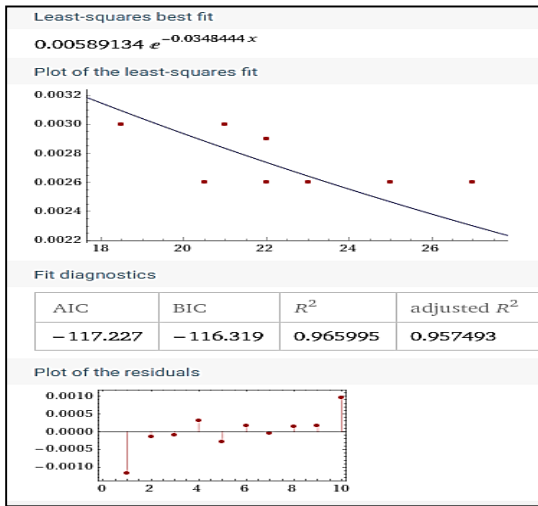


Figure 12. Regression plot of A_v with plastic limit

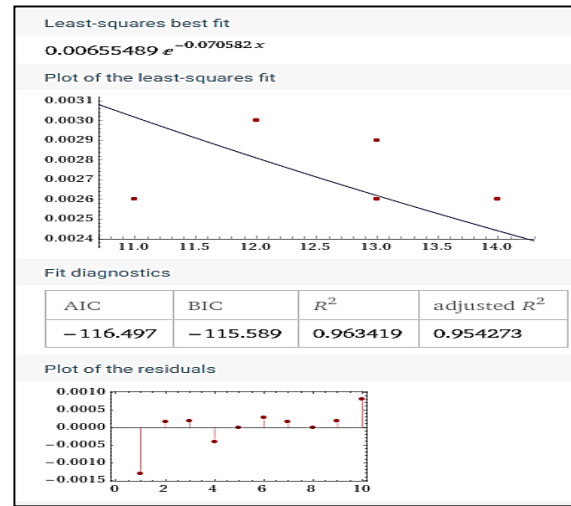


Figure 13. Regression plot of A_v with plastic index

4. Conclusions

From the study, the following findings are presented in order to underscore the necessity of the research work:

- The soil experimented upon were all classified according to AASHTO and USCS as A-2-6 and SC respectively, having liquid limit values greater than 30%, plastic limit values less than 30% and Plasticity Index of less than 20%.
- The Coefficient of Determination of the models for the ten samples have values above 90%, further indicating the adequacy of the model in Prediction. Also, models formulated were fit and demonstrates that the influence of compressibility indices is associated to the Atterberg Limits LL, PL and PI.
- In light of the close agreement between the estimated observed compression index and coefficient of consolidation values, additional investigation into the

effects of other soil characteristics on these values is encouraged.

Nomenclature

AASHTO – American Association of State Highway and Transport Officials
USCS – Unified Soil Classification System
LL - Liquid limit
PL - Plastic Limit
PI - Plasticity Index
 C_c - Compression Index
 C_v - Coefficient of Consolidation
 A_v - Coefficient of Compressibility
 M_v - Coefficient of Volume Compressibility
 R^2 – Coefficient of Determination

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Biographical notes



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